

Directions: Calculators are allowed. Show all your working! Use the back of the page if you run out of space.

1. (5 marks) Find $\int x\sqrt{x} - 7\sin(x) dx$.

$$\begin{aligned} & \int x\sqrt{x} - 7\sin(x) dx \\ &= \int x^1 x^{1/2} - 7\sin(x) dx \\ &= \int x^{3/2} - 7\sin(x) dx \end{aligned}$$

$$\begin{aligned} &= \frac{x^{5/2}}{5/2} + 7\cos(x) + C \\ &= \frac{2}{5} x^{5/2} + 7\cos(x) + C \end{aligned}$$

2. (5 marks) Find $\int (3x^2 - 1)^2 dx$. (Hint: expand the brackets first!)

$$\begin{aligned} & \int (3x^2 - 1)^2 dx \\ &= \int 9x^4 - 6x^2 + 1 dx \end{aligned}$$

$$\begin{aligned} &= \frac{9x^5}{5} - \frac{6x^3}{3} + x + C \\ &= \frac{9x^5}{5} - 2x^3 + x + C \end{aligned}$$

3. (5 marks) Find $\int x^2 e^{x^3} dx$.

$$\begin{aligned} \text{Let } u &= x^3 \\ \frac{du}{dx} &= 3x^2 \\ du &= 3x^2 dx \end{aligned}$$

$$\begin{aligned} & \int x^2 e^{x^3} dx \\ &= \frac{1}{3} \int 3x^2 e^{x^3} dx \\ &= \frac{1}{3} \int e^u du = \frac{1}{3} e^u + C = \frac{1}{3} e^{x^3} + C \end{aligned}$$

4. (5 marks) Using the substitution $u = x^2 + 2x$, find

$$\begin{aligned} u &= x^2 + 2x \\ \frac{du}{dx} &= 2x + 2 \\ du &= 2(x+1) dx \end{aligned}$$

$$\begin{aligned} & \int \frac{x+1}{(x^2+2x)^3} dx \\ &= \frac{1}{2} \int \frac{2(x+1)}{(x^2+2x)^3} dx \\ &= \frac{1}{2} \int \frac{1}{u^3} du \\ &= \frac{1}{2} \int u^{-3} du \\ &= \frac{1}{2} \frac{u^{-2}}{-2} + C = -\frac{1}{4} (x^2+2x)^{-2} + C \end{aligned}$$