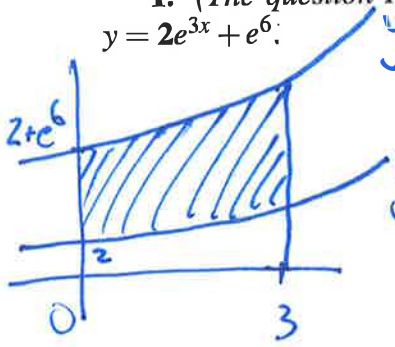
Math 20
Name:

Quiz 4
Date: 2/12/2014

OK, so I stuffed question 1 up a bit here. There was a typo in the version you did in class, which made my 'hints' a little bit, well, completely wrong. Here's the (easier) question I wanted you to do, and then how to do the (harder) question that you actually got in class. Sorry!

1. *(The question I wanted to give you)* Find the area between the curves $x = 0$, $x = 3$, $y = 2e^{3x}$ and $y = 2e^{3x} + e^6$.



$y = 2e^{3x} + e^6$

$y = 2e^{3x}$

$$\text{Area} = \int_0^3 [2e^{3x} + e^6] - [2e^{3x}]\,dx$$

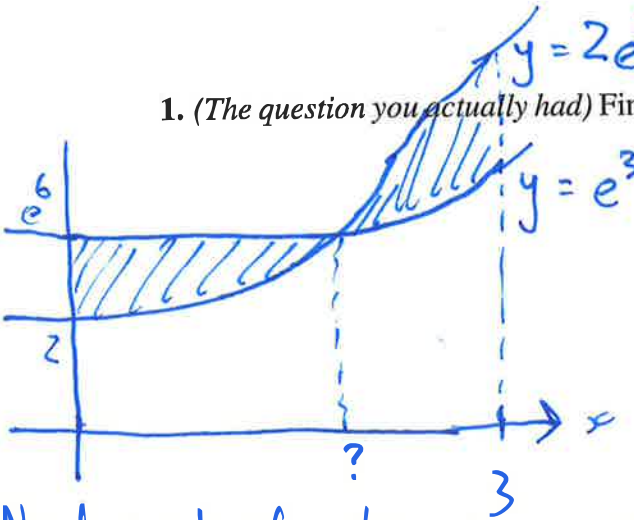
$$= \int_0^3 2e^{3x} + e^6 - 2e^{3x}\,dx$$

$$= \int_0^3 e^6\,dx$$

$$= e^6 x \Big|_0^3$$

$$= 3e^6 - 0.$$

1. *(The question you actually had)* Find the area between the curves $x = 0$, $x = 3$, $y = 2e^{3x}$ and $y = e^{3x} + e^6$.



Need point of intersection, ie solve

$$2e^{3x} = e^{3x} + e^6$$

$$e^{3x} = e^6$$

$$3x = 6$$

$$\boxed{x = 2}$$

So

$$\text{Area} = \int_0^2 e^{3x} + e^6 - 2e^{3x}\,dx + \int_2^3 2e^{3x} - (e^{3x} + e^6)\,dx$$

$$= \int_0^2 e^6 - e^{3x}\,dx + \int_2^3 e^{3x} - e^6\,dx$$

$$= \left[e^6 x - \frac{e^{3x}}{3}\right]_0^2 + \left[\frac{e^{3x}}{3} - e^6 x\right]_2^3$$

$$= \left(2e^6 - \frac{e^6}{3}\right) - \left(0 - \frac{1}{3}\right)$$

$$+ \left(\frac{e^9}{3} - 3e^6\right) - \left(\frac{e^6}{3} - 2e^6\right)$$

$$= e^9/3 + \frac{e^6}{3} - \frac{1}{3}$$

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Directions: Calculators are allowed, but you shouldn't need to use your calculator! Show all your working, neatly! Use the back of the page if you run out of space.

1. *(5 marks)* Find the area between the curves $x = 0$, $x = 3$, $y = 2e^{3x}$ and $y = e^{3x} + e^{6}$.

See my other explanation ...

2. *(5 marks)* Find $\int (4x - 12)e^{-8x}dx$ using integration by parts.

Column method:

D	I
$4x-12$	e^{-8x}
4	$e^{-8x}/-8$
0	$e^{-8x}/64$

(+ , −)

So $\int (4x-12)e^{-8x}dx = (4x-12)\left[\frac{e^{-8x}}{-8}\right] - \frac{4e^{-8x}}{64}$ +c

Formula method:

Let $u = 4x-12$, $u' = 4$; $v = e^{-8x}/-8$, $v' = e^{-8x}$

So $\int (4x-12)e^{-8x}dx = uv - \int vu'dx$

$= -\frac{(4x-12)e^{-8x}}{8} + \frac{4}{8}\int e^{-8x}dx$

$= -\frac{(4x-12)e^{-8x}}{8} + \frac{1}{2}\frac{e^{-8x}}{-8} + C$

3. *(5 marks)* Find $\int x\sqrt{x+1}dx$.

First let's note that $\int (x+1)^n dx = \frac{(x+1)^{n+1}}{n+1}$, if we use integration by substitution with $u = x+1$. (Check this!).

∴, let $u = x$, $v = \frac{(x+1)^{3/2}}{3/2}$

$u' = 1$, $v' = (x+1)^{1/2}$

So $\int x\sqrt{x+1}dx = uv - \int vu'dx$

$= \frac{2}{3}x(x+1)^{3/2} - \frac{2}{3}\int (x+1)^{3/2}dx$

$= \frac{2}{3}x(x+1)^{3/2} - \frac{2}{3}\frac{(x+1)^{5/2}}{5/2} + C$

$= \frac{2}{3}x(x+1)^{3/2} - \frac{4}{15}(x+1)^{5/2} + C.$