Math 20 **Quiz 8**

Name: **Date: 3/26/2014**

Directions: Calculators are allowed, but you shouldn't need to use your calculator. Use your equals signs! Use the back of the page if you run out of space.

(10 marks) Find the minimum value of the function $f(x,y) = x^2 + 2x + 9y^2 + 4y + 8xy$, subject to the constraint that the solution must lie on the plane $x + y = 1$.
(a) Write down an appropriate constraint $g(x,y) = 0$.
(b) Form the Lagrange function $F(x,y,\lambda) = f(x,y) - \lambda g(x,y)$.
(c) Solve the appropriate set of simultaneous equations that arise from differentiating this function to find the optimal values for x and y.

(a) $f = x^2 + 2x + 9y^2 + 4y + 8xy$

$g = x + y - 1 = 0$ constraint.

(b) so $F = x^2 + 2x + 9y^2 + 4y + 8xy - \lambda x - \lambda y + \lambda$

(c) Let $F_x = 2x + 2 + 8y - \lambda = 0$ ... (1)

$F_y = 18y + 4 + 8x - \lambda = 0$ ... (2)

$F_\lambda = -x - y + 1 = 0.$ ... (3)

(1): $\lambda = 2x + 2 + 8y$

(2): $\lambda = 18y + 4 + 8x$

so $2x + \cancel{2} + 8y = 18y + \overset{2}{\cancel{4}} + 8x$

$-10y = 2 + 6x$

$y = \dfrac{-(2+6x)}{10}$

$= \dfrac{-(1+3x)}{5}.$

so sub into (3):

$x + y = 1$

$x - \dfrac{(1+3x)}{5} = 1$

$5x - 1 - 3x = 5$

$2x = 6$

$x = 3$

so $y = 2.$

$\boxed{\text{Solutions } (x,y) = (3,2)}$